5.6/15)

$$N = 5000\left(25 + t e^{-t/20}\right) \quad 0 \le t \le 100$$

$$\frac{dN}{dt} = 5000\left(e^{-t/20} + t\left(-\frac{1}{20}\right)e^{-t/20}\right) = 0$$

s.c. of $5000 e^{-t/20}\left(1 - \frac{t}{20}\right)$

$$e^{-t/20} - \frac{t}{20} e^{-t/20} = 0$$

$$e^{-t/20}\left[1 - \frac{t}{20}\right] = 0$$

$\therefore t = 20$ crit. #

+ + + | − − −

20

rel max at $t = 20$

$\therefore$ abs max on $[0,100]$

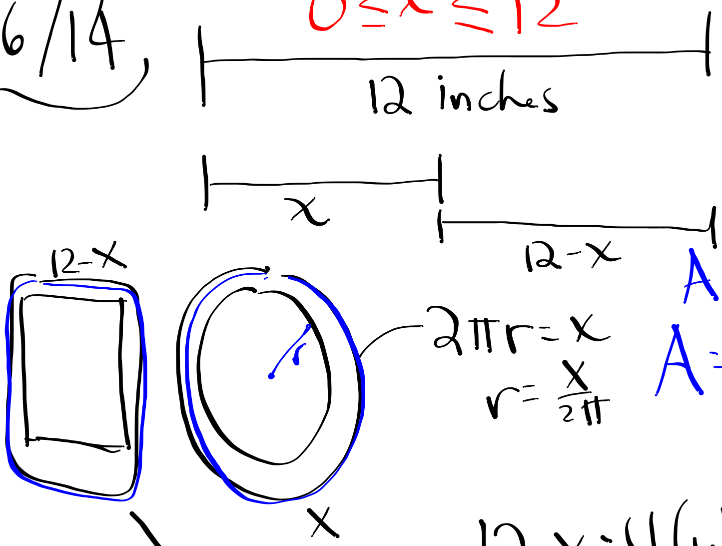
ABS MIN on $[0,100]$

$$N(t=0) = 5000\left(25 + 0 e^{-0/20}\right) = 5000 \cdot 25$$

$$N(t=100) = 5000\left(25 + 100 e^{-5}\right)$$

5.6/14,

$0 \le x \le 12$



$A = \pi r^2$

$2\pi r = x$

$r = \frac{x}{2\pi}$

$A = \pi\left(\frac{x^2}{4\pi^2}\right)$

$12 - x = 4(y)$

$A = y^2 = \left(\frac{12-x}{4}\right)^2$

total area $= \frac{x^2}{4\pi} + \left(\frac{12-x}{4}\right)^2$

$\frac{2x}{4\pi}$

$(TA)' = \frac{x}{2\pi} + 2\left(\frac{12-x}{4}\right)(-1)$

$= \frac{x}{2\pi} - 6 + \frac{x}{2} = 0$

$= \frac{(1+\pi)x - 12\pi}{2\pi} = 0$

$\therefore (1+\pi)x = 12\pi$

$x = \frac{12\pi}{(1+\pi)}$

gives me a minimum

$- \; - \; - \; | \; + \; + \; +$

$\frac{12\pi}{(1+\pi)}$

$f(x) = \frac{x^2}{4\pi}$ what $f'(x)$

using QR

$$f'(x) = \frac{2x(4\pi) - x^2(0)}{(4\pi)^2} = \frac{2x}{4\pi}\left(\frac{4\pi}{4\pi}\right)$$

$$= \frac{2x}{4\pi} = \frac{x}{2\pi}$$

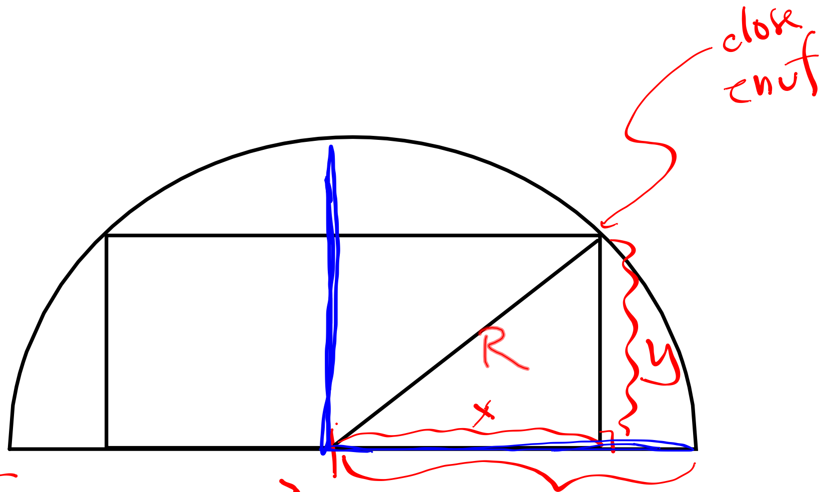
$$\frac{d}{dx}\left(\frac{f(x)}{k}\right) = \frac{f'(x)\cdot k - f(x)\cdot 0}{k^2}$$

$$= \frac{f'(x)\cdot k}{k\,k} = \frac{f'(x)}{k}$$

$$\parallel$$

$$\frac{d}{dx}\left(\frac{1}{k}\cdot f(x)\right)$$

$$= \frac{1}{k} f'(x)$$



$$R^2 = x^2 + y^2 \rightarrow y = \sqrt{R^2 - x^2}$$

$$A = 2xy$$

$$A = 2x(R^2 - x^2)^{1/2}$$

$$A' = 2\left[(R^2-x^2)^{1/2} + \frac{x}{2}(-2x)\left(\frac{1}{\sqrt{R^2-x^2}}\right)\right]$$

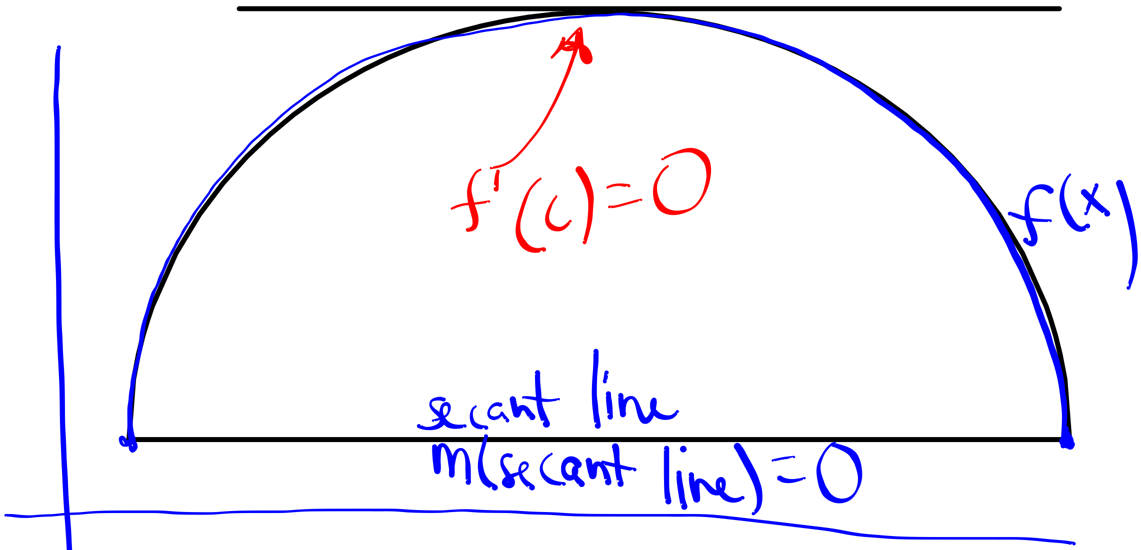
$$\left(\sqrt{R^2-x^2}\right)\left[\sqrt{R^2-x^2} - \frac{x^2}{\sqrt{R^2-x^2}}\right] = 0\left[\sqrt{R^2-x^2}\right]$$

$$(R^2 - x^2) - x^2 = 0$$

$$R^2 - 2x^2 = 0$$

$$x = \frac{R}{\sqrt{2}}$$

5.8 Mean Value Theorem



What f^n properties ENSURE

$\exists c \ni f'(c)=0 \Rightarrow$ derivative exists

$\Rightarrow$ continuous...

MVT

if $f(x)$ is:

* continuous on $[a,b]$
* differentiable on (a,b)

then

there is a c (where $a<c<b$)

such that $f'(c) = \frac{f(b)-f(a)}{b-a}$

end if

HW

58/ 1-15 odd

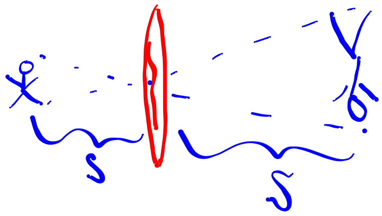
5.6/ 16-21

3.7/43) $\dfrac{1}{s}+\dfrac{1}{S}=\dfrac{1}{f}$

s = object distance from lens

S = image distance from lens

f = focal length



a certain lens . . .

$f = 6 \text{ cm}$

$\dfrac{ds}{dt} = -2 \dfrac{\text{cm}}{\text{sec}}$

what is $\dfrac{dS}{dt}$ when $s = 10 \text{ cm}$

(2) $\dfrac{1}{s}+\dfrac{1}{S}=\dfrac{1}{6}$

$s^{-1}+S^{-1}=6^{-1}$

(3) $-\dfrac{1}{s^2}\dfrac{ds}{dt}-\dfrac{1}{S^2}\dfrac{dS}{dt}=0$

$-\dfrac{1}{10^2}(-2)-\dfrac{1}{\boxed{15}^2}\dfrac{dS}{dt}=0$ $\qquad \dfrac{1}{10}+\dfrac{1}{S}=\dfrac{1}{6}$

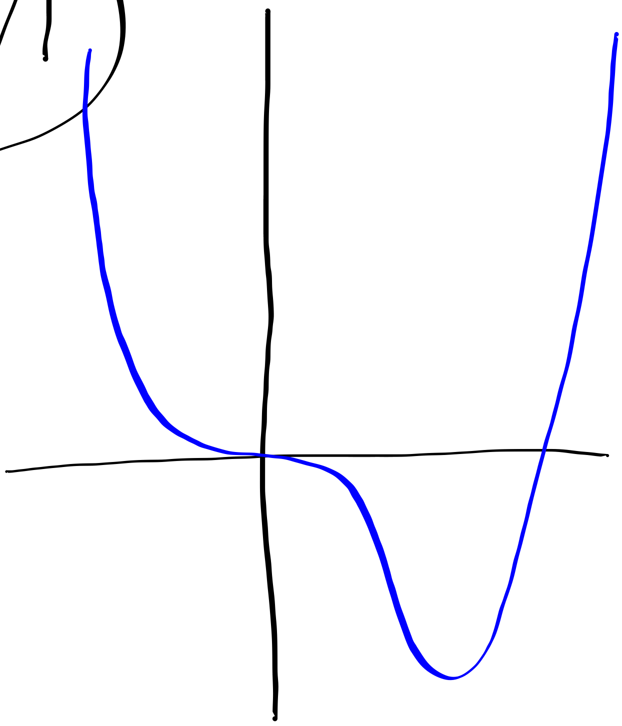
$\dfrac{1}{50}-\dfrac{1}{225}\dfrac{dS}{dt}=0$ $\qquad \dfrac{6S+60}{60S}=\dfrac{10S}{60S}$

$6S+60=10S$

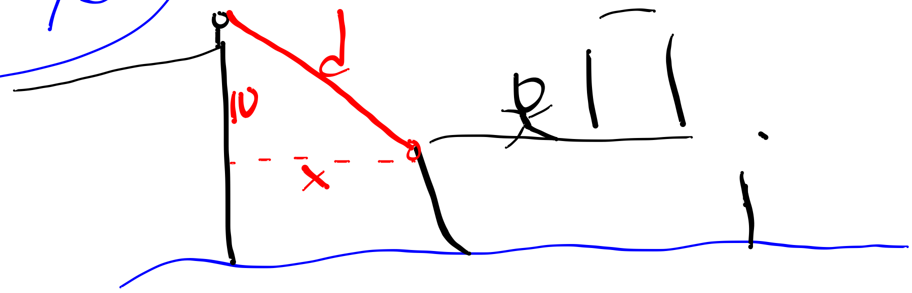
$60=4S$

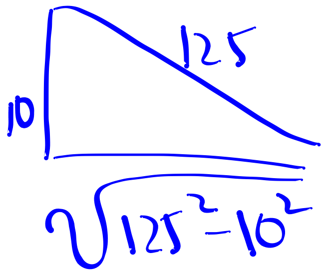
$\dfrac{dS}{dt}=\dfrac{225}{50}=\dfrac{9}{2}\text{ cm/sec}$ $\qquad \therefore S=15$

5.8/1)



3.7/31





② $10^2+x^2=d^2$

$\cancel{2}x\frac{dx}{dt}=\cancel{2}d\frac{dd}{dt}$

$-12\,\frac{ft}{min}$ $\quad$ $125\,ft$

$$\left(\sqrt{125^2-10^2}\right)(-12)=125\frac{dd}{dt}$$

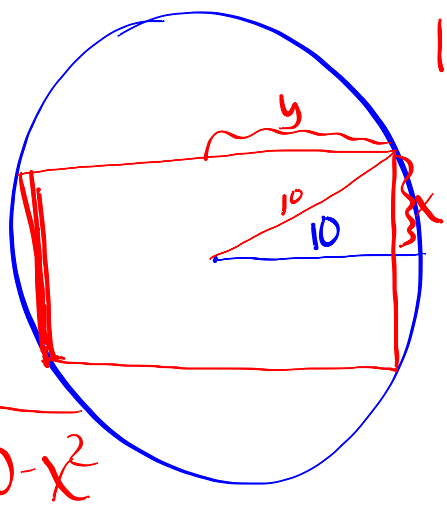
$$\frac{-12\sqrt{125^2-10^2}}{125}=\frac{dd}{dt} \qquad \left[\frac{ft}{min}\right]$$

$$=\frac{-12\cdot 5\sqrt{25^2-2^2}}{125}$$

$$=-\frac{12}{25}\sqrt{621}\ \ ft/min$$

5.6) 9

$A = 4x \cdot y$



$10^2 = x^2 + y^2$

$y = \sqrt{100-x^2}$

$A = 4x\sqrt{100-x^2}$

$A' = 4\left[\sqrt{100-x^2} + x\frac{-2x}{2\sqrt{100-x^2}}\right]$

$A' = 0 \Rightarrow$

$\sqrt{100-x^2} - \frac{x^2}{\sqrt{100-x^2}} = 0$ $\quad \frac{\sqrt{100-x^2}}{\sqrt{100-x^2}} \cdot \sqrt{100-x^2} - \frac{x^2}{\sqrt{100-x^2}}$

$\frac{(\sqrt{100-x^2})^2 - x^2}{\sqrt{100-x^2}} = 0$ so $100 - 2x^2 = 0$

$(100-x^2)-x^2$

$x = \sqrt{50}$

$y = \sqrt{50}$

dimensions: $2\sqrt{50} \times 2\sqrt{50}$

Picture

↓

Eqn Always true

Related Rates

deriv w/x

eqn 1 variable

Optimization (find max/min value)

deriv w/x

$x^{\pm 5/3}$

$(x^{1/3})^5$

1 3 kids = 13

2 ~~Multiplied~~ —

Go ~~hous~~ # next
Back door

3 Oldest kid
play piano

$11+1+1=13$

11

$2+10+1$

20

$8+4$

$3+9+1$

27

$9\,2\,2$

36

$\cancel{9+2+2}$

$\cancel{36}$

$8+3+2$

48

$8+4+1$

32

$7+3+3$

63

$7+4+2$

56

$7+5+1$

35

$\cancel{6+6+1}$

$\cancel{36}$

$6+5+2$

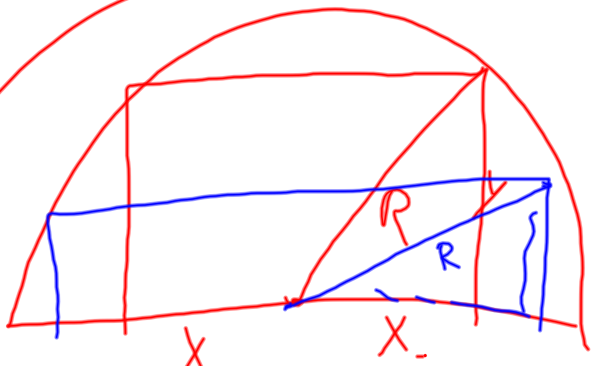
60

$6+4+3$

72

$5+4+4$

80.



$$A = 2xy$$

$$y = \sqrt{R^2 - x^2}$$

$$(R^2 - x^2)^{\frac{1}{2}}$$

$$\frac{1}{2}(R^2 - x^2)^{-\frac{1}{2}}(2x + 2R)$$

$$A = 2x\left(\sqrt{R^2 - x^2}\right)$$

$$A' = 2\left[\left(\sqrt{R^2 - x^2}\right) + (x)\left(\frac{1}{2\sqrt{R^2 - x^2}}(-2x)\right)\right] = 0$$

$$-R^2 + x^2 \qquad = \frac{-2x^2 + 2xR^2}{2\sqrt{R^2 - x^2}}$$

$$x = \sqrt{R^2 - x^2}$$

$$R^2 - x^2 = x^2$$

$$R^2 = 2x^2$$

$$\frac{R^2}{2} = x^2$$

$$\pm\frac{R}{\sqrt{2}} = \pm\sqrt{\frac{R^2}{2}} = x$$

$$x = \sqrt{R^2 - \left(\sqrt{\frac{R^2}{2}}\right)^2}$$

$$y = \sqrt{R^2 - \frac{R^2}{2}}$$

$$x = \sqrt{\frac{R^2}{2}}$$

$C = (2x) + (4y)$

$3200 = xy$

$\frac{3200}{y} = x$

$x = \frac{3200}{40}$

$x = 80$

$C = \left(2\left(\frac{3200}{\cancel{y}}\right)\right) + 4y$

$C = \frac{6400}{y} + 4y$

$C = \frac{6400 + 4y^2}{y}$

$C' = \frac{(8y)(y) - (6400 + 4y^2)}{y^2}$

$0 = \frac{8x^2 - 6400 - 4x^2}{4y^2 - 6400 \over x^2}$

$4y^2 = 6400$

$x^2 = 1600$

$y = 40$